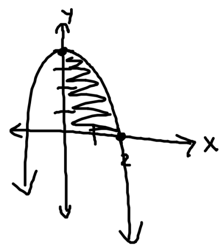
DO NOW

pg 305; 73 $f(x) = 4 - x^2 \quad [0, 2]$



5.3 Riemann Sums and Definite Integrals

Riemann Sums: The area summations in Section 5.2 are regular partition (uniform interval) reimann sums.

* However, intervals do not have to be uniform.

$\|\Delta\|$ is: the norm of the partitions
↳ width of the largest subinterval.

$$\|\Delta\| = \Delta x = \frac{b-a}{n} \quad \text{(uniform intervals)}$$

Definition of a Definite Integral

If f is defined on $[a, b]$ and the limit exists then

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)(\Delta x_i) = \int_a^b f(x)\,dx$$

a = lower limit of integration

b = upper limit of integration

c_i = any point on i^{th} subinterval $(a + \Delta x\, i)$

Δx_i = the width of i^{th} subinterval $\left(\frac{b-a}{n}\right)$

***NOTE: $\|\Delta\| \to 0$ implies $n \to \infty$

*A definite integral is NOT the same as an indefinite integral!!!
↳ number ↳ family of functions

Definite Integrals can be positive, negative or zero.

**To be interpreted strictly as area - $f(x)$ must be nonnegative for all x in the interval.

Examples: page 314; 9 - 14

9. $\int_{-1}^{5} (3x+10)\,dx$

10. $\int_{0}^{4} 6x(4-x)^2\,dx$

Example: Evaluate the definite integral by the limit definition.

1. $\int_{-2}^{1} 2x\,dx = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x$

$[-2, 1]$

$\Delta x = \frac{1 - -2}{n} = \frac{3}{n}$

$c_i = a + \Delta x\, i$
$\quad -2 + \frac{3i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} 2\left(-2 + \frac{3i}{n}\right)\frac{3}{n}$

$\lim_{n\to\infty} \frac{6}{n}\left[\sum_{i=1}^{n} -2 + \frac{3i}{n}\right]$

$\lim_{n\to\infty} \frac{6}{n}\left[\sum_{i=1}^{n} -2 + \frac{3}{n}\sum_{i=1}^{n} i\right]$

$\lim_{n\to\infty} \left[\frac{6}{\cancel{n}} \cdot -2\cancel{n} + \frac{\cancel{18}^{\,9}}{\cancel{n^2}_{\,n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}_{\,1}}\right]$

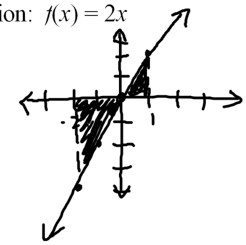
$\lim_{n\to\infty} \left(-12 + \frac{9n+9}{n}\right)$

$\lim_{n\to\infty} \left(-12 + 9 + \frac{9}{n}\right)$

$-12 + 9$

$\boxed{-3}$

Let's look at the graph of this function: $f(x) = 2x$



From $[0,1]$

$A = \frac{1}{2}bh$
$= \frac{1}{2}(1)(2)$
$= 1$

From $[-2,0]$

$A = \frac{1}{2}bh$
$\frac{1}{2}(2)(4)$
$= 4$

* Below x-axis is like "negative area"

$1 - 4 = \boxed{-3}$

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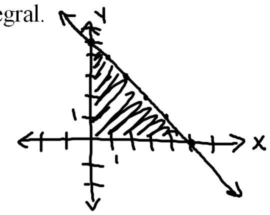
Basic formulas can be used to calculate definite integrals instead of using the limit of a summation.

Examples: Sketch the graph of the function. Then use a geometric formula to evaluate the integral.

2. $\int_0^5 (5 - x)dx$



$A = \frac{1}{2}bh$
$\frac{1}{2}(5)(5)$
12.5

$\therefore \int_0^5 (5-x)dx = 12.5$

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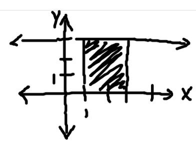
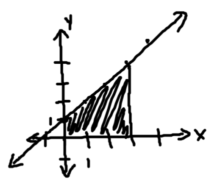
3. $\int_1^3 3dx$

$A = lw$
$= 2(3)$
6



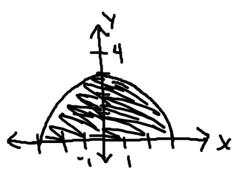
4. $\int_0^3 (x + 1)dx$

$A = \frac{1}{2}h(b_1 + b_2)$
$\frac{1}{2}(3)(1+4)$
$\frac{3}{2}(5) = 7.5$



5. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

$A = \frac{1}{2}\pi r^2$
$\frac{1}{2}\pi(3)^2$
$\boxed{\frac{9\pi}{2}}$



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HOMEWORK

pg 314 - 315; 5, 7, 15 - 21, 23, 25, 27

#21 - notice variable change...

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